INSTITUTO TECNOLÓGICO SUPERIOR DE LERDO
LABORATORIO DE MATEMÁTICAS

Formulario de Matemáticas

DESIGUALDADES Y VALOR ABSOLUTO

$Si \quad a<b \quad entonces \quad a+c<b+c$

$Si \quad a<b \quad y \quad c>0 \quad entonces \quad ac<bc$

$Si \quad a<b \quad y \quad c<0 \quad entonces \quad ac>bc$

$$|a|=\begin{cases} a & si \quad a\geq 0 \\ -a & si \quad a<0 \end{cases} \qquad |a|=|-a|$$

$|a|=b \quad si \quad y \quad solo \quad si \quad a=b \quad ó \quad a=-b$

$|a|<b \quad si \quad y \quad solo \quad si \quad -b<a<b$

$|a|>b \quad si \quad y \quad solo \quad si \quad a>b \quad ó \quad a<-b$

$|ab|=|a||b| \qquad \left|\frac{a}{b}\right|=\frac{|a|}{|b|}$

$|a+b|\leq|a|+|b| \qquad |a-b|\geq|a|-|b|$

$\sqrt{a^2}=|a| \qquad |a|^2=a^2$

$$|a|<|b| \quad \Leftrightarrow \quad a^2<b^2$$

EXPONENTES

$a^0=1 \qquad \frac{1}{a^p}=a^{-p}$

$a^p\cdot a^q=a^{p+q} \qquad \frac{a^p}{a^q}=a^{p-q}$

$\left(a^p\right)^q=a^{pq} \qquad (a\cdot b)^p=a^p\cdot b^p$

$\left(\frac{a}{b}\right)^p=\frac{a^p}{b^p} \qquad a^{p/q}=\sqrt[q]{a^p}$

LOGARITMOS

$e^{\ln x}=x \qquad \ln x^n=n\ln x$

$\ln(ax)=\ln a+\ln x \qquad \ln\frac{x}{a}=\ln x-\ln a$

$Log_aN=x\Rightarrow a^x=N \qquad Log_{10}N=LogN$

$$Log_eN=\ln N$$

TRIÁNGULO DE PASCAL

```
          1
        1 2 1
      1 3 3 1
    1 4 6 4 1
  1 5 10 10 5 1
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$$(a+b)^n=a^n+na^{n-1}b+\frac{n(n-1)}{2!}a^{n-2}b^2+\frac{n(n-1)(n-2)}{3!}a^{n-3}b^3+\ldots+b^n$$

ALGUNOS PRODUCTOS

$a\cdot(c+d)=ac+ad$

$(a+b)(a-b)=a^2-b^2$

$(a+b)(a+b)=(a+b)^2=a^2+2ab+b^2$

$(a-b)(a-b)=(a-b)^2=a^2-2ab+b^2$

$(x+b)(x+d)=x^2+(b+d)x+bd$

$(ax+b)(cx+d)=acx^2+(ad+bc)x+bd$

$(a+b)(c+d)=ac+ad+bc+bd$

$(a+b)^3=a^3+3a^2b+3ab^2+b^3$

$(a-b)^3=a^3-3a^2b+3ab^2-b^3$

$(a+b+c)^2=a^2+b^2+c^2+2ab+2ac+2bc$

$$(a-b)(a^2+ab+b^2)=a^3-b^3$$

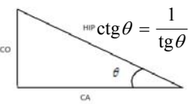
TRIGONOMETRÍA

$\operatorname{sen}\theta=\frac{CO}{HIP} \qquad \cos\theta=\frac{CA}{HIP}$

$\tan\theta=\frac{\operatorname{sen}\theta}{\cos\theta}=\frac{CO}{CA} \qquad \csc\theta=\frac{1}{\operatorname{sen}\theta}$

$\sec\theta=\frac{1}{\cos\theta} \qquad \operatorname{ctg}\theta=\frac{1}{\operatorname{tg}\theta}$

π radianes $=180°$



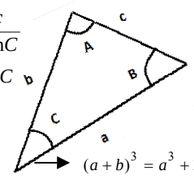
Valores de ángulos comunes

θ	sen	cos	tg	ctg	sec	csc
0°	0	1	0	∞	1	∞
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$
90°	1	0	∞	0	∞	1

$\frac{a}{\operatorname{sen}A}=\frac{b}{\operatorname{sen}B}=\frac{c}{\operatorname{sen}C}$

$c^2=a^2+b^2-2ab\cos C$

$\frac{a+b}{a-b}=\frac{\tan\frac{1}{2}(A+B)}{\tan\frac{1}{2}(A-B)}$



$(a+b)^3=a^3+3a^2b+3ab^2+b^3$

IDENTIDADES TRIGONOMÉTRICAS

$\operatorname{sen}^2\theta+\cos^2\theta=1 \qquad \operatorname{sen}\theta\csc\theta=1$

$\sec^2\theta-\operatorname{tg}^2\theta=1 \qquad \cos\theta\sec\theta=1$

$\csc^2\theta-\operatorname{ctg}^2\theta=1 \qquad \tan\theta\operatorname{ctg}\theta=1$

$\tan\theta=\frac{\operatorname{sen}\theta}{\cos\theta} \qquad ctg\theta=\frac{\cos\theta}{\operatorname{sen}\theta}$

$\operatorname{sen}(-\theta)=-\operatorname{sen}\theta \qquad \cos(-\theta)=\cos\theta$

$$\tan(-\theta)=-\tan\theta$$

$\operatorname{sen}^2\theta=\frac{1}{2}-\frac{1}{2}\cos 2\theta$

$\cos^2\theta=\frac{1}{2}+\frac{1}{2}\cos 2\theta$

$tg^2\theta=\frac{1-\cos 2\theta}{1+\cos 2\theta}$

$\operatorname{sen}2\theta=2\operatorname{sen}\theta\cos\theta$

$\cos 2\theta=\cos^2\theta-\operatorname{sen}^2\theta$

$\operatorname{tg}2\theta=\frac{2\operatorname{tg}\theta}{1-\operatorname{tg}^2\theta}$

$\operatorname{sen}(\alpha\pm\beta)=\operatorname{sen}\alpha\cos\beta\pm\cos\alpha\operatorname{sen}\beta$

$\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\operatorname{sen}\alpha\operatorname{sen}\beta$

$\operatorname{tg}(\alpha\pm\beta)=\frac{\operatorname{tg}\alpha\pm\operatorname{tg}\beta}{1\mp\operatorname{tg}\alpha\operatorname{tg}\beta}$

$\operatorname{sen}\frac{\theta}{2}=\pm\sqrt{\frac{1-\cos\theta}{2}}$

$\cos\frac{\theta}{2}=\pm\sqrt{\frac{1+\cos\theta}{2}}$

$\operatorname{sen}\alpha\operatorname{sen}\beta=\frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$

$\operatorname{sen}\alpha\cos\beta=\frac{1}{2}\left[\cos(\alpha-\beta)+\operatorname{sen}(\alpha+\beta)\right]$

$\cos\alpha\cos\beta=\frac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right]$

FUNCIONES HIPERBÓLICAS

$senhx=\frac{e^x-e^{-x}}{2} \qquad coshx=\frac{e^x+e^{-x}}{2}$

$tghx=\frac{senhx}{\cosh x}=\frac{e^x-e^{-x}}{e^x+e^{-x}}$

NÚMEROS COMPLEJOS

Definición: $a+bi \quad$ donde $a,b\in\mathbb{R}$

$i=\sqrt{-1} \qquad i^2=-1 \qquad i^3=-i \qquad i^4=1$

$(a+bi)+(c+di)=(a+c)+(b+d)i$

$(a+bi)-(c+di)=(a-c)+(b-d)i$

$(a+bi)(c+di)=(ac-bd)+(ad+bc)i$

$\frac{(a+bi)}{(c+di)}=\frac{(a+bi)}{(c+di)}\cdot\frac{(c-di)}{(c-di)}$

$$=\frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$$

$e^{i\theta}=\cos\theta-i\operatorname{sen}\theta$

CUADRÁTICA

$x^2-A^2=0 \quad \rightarrow \quad x=\pm\sqrt{A^2}=\pm A$

$Ax^2+Bx+C=0$

$\Rightarrow \quad x_{1,2}=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$

$B^2-4AC=$ Discriminante

LÍMITES

$\lim_{x\to c}k=k$

$\lim_{x\to c}x=c$

$\lim_{x\to c}k\,f(x)=k\lim_{x\to c}f(x)$

$\lim_{x\to c}\left[f(x)\pm g(x)\right]=\lim_{x\to c}f(x)\pm\lim_{x\to c}g(x)$

$\lim_{x\to c}\left[f(x)\right]^n=\left[\lim_{x\to c}f(x)\right]^n$

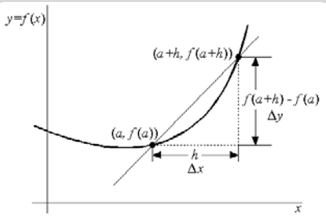
$\lim_{x\to 0}(1+x)^{\frac{1}{x}}=e=2.71828\ldots$

$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e \qquad \lim_{x\to 0}\frac{\operatorname{sen}x}{x}=1$

$\lim_{x\to 0}\frac{1-\cos x}{x}=0 \qquad \lim_{x\to 0}\frac{e^x-1}{x}=1$

$\lim_{x\to 1}\frac{x-1}{\ln x}=1$

DERIVADAS



$D_x f(x)=\frac{df}{dx}=Lim_{\Delta x\to 0}\frac{f(x+\Delta x)}{\Delta x}=Lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}$

$\frac{d}{dx}(c)=0$

$\frac{d}{dx}(cx)=c$

$\frac{d}{dx}(cx^n)=cx^{n-1}$

$\frac{d}{dx}(u\pm v\pm w\pm\ldots)=\frac{du}{dx}\pm\frac{dv}{dx}\pm\frac{dw}{dx}\pm\ldots$

$\frac{d}{dx}(cu)=c\frac{du}{dx}$

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$$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$$

$$\frac{d}{dx}(uvw)=uv\frac{dw}{dx}+uw\frac{dv}{dx}+vw\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\left(du/dx\right)-u\left(dv/dx\right)}{v^2}$$

$$\frac{d}{dx}(u^n)=nu^{n-1}\frac{du}{dx}$$

$$\frac{dF}{dx}=\frac{dF}{du}\cdot\frac{du}{dx}$$ (Regla de la Cadena)

$$\frac{du}{dx}=\frac{1}{dx/du}$$

$$\frac{dF}{dx}=\frac{dF/du}{dx/du}$$

$$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{f'_2(t)}{f'_1(t)}$$ donde $\begin{cases}x=f_1(t)\\y=f_2(t)\end{cases}$

DERIVADAS DE FUNCIONES LOG Y EXP

$$\frac{d}{dx}(\ln u)=\frac{1}{u}\cdot\frac{du}{dx}$$

$$\frac{d}{dx}(\log u)=\frac{\log e}{u}\cdot\frac{du}{dx}$$

$$\frac{d}{dx}(\log_a u)=\frac{\log_a e}{u}\cdot\frac{du}{dx}\quad a>0,\ a\neq 1$$

$$\frac{d}{dx}(e^u)=e^u\cdot\frac{du}{dx}$$

$$\frac{d}{dx}(a^u)=a^u\ln a\cdot\frac{du}{dx}$$

$$\frac{d}{dx}(u^v)=vu^{v-1}\frac{du}{dx}+\ln u\cdot u^v\frac{dv}{dx}$$

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DERIVADAS DE FUNCIONES TRIGONOMÉTRICAS

$$\frac{d}{dx}(\operatorname{sen} u)=\cos u\frac{du}{dx}$$

$$\frac{d}{dx}(\cos u)=-\operatorname{sen} u\frac{du}{dx}$$

$$\frac{d}{dx}(\tan u)=\sec^2 u\frac{du}{dx}$$

$$\frac{d}{dx}(\cot u)=-\csc^2 u\frac{du}{dx}$$

$$\frac{d}{dx}(\sec u)=-\sec u\tan u\frac{du}{dx}$$

$$\frac{d}{dx}(\csc u)=-\csc u\cot u\frac{du}{dx}$$

DERIVADAS DE FUNCIONES TRIGONOMÉTRICAS INVERSAS

$$\frac{d}{dx}(\arcsin u)=\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}(\arccos u)=-\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}(\arctan u)=\frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}(\operatorname{arccot} u)=-\frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}(\operatorname{arcsec} u)=\pm\frac{1}{u\sqrt{u^2-1}}\frac{du}{dx}\quad\begin{cases}+\text{ si } u>1\\-\text{ si } u<-1\end{cases}$$

$$\frac{d}{dx}(\operatorname{arccsc} u)=\mp\frac{1}{u\sqrt{u^2-1}}\frac{du}{dx}\quad\begin{cases}-\text{ si } u>1\\+\text{ si } u<-1\end{cases}$$

DERIVADAS DE FUNCIONES HIPERBÓLICAS

$$\frac{d}{dx}(\operatorname{senh} u)=\cosh u\frac{du}{dx}$$

$$\frac{d}{dx}(\cosh u)=\operatorname{senh} u\frac{du}{dx}$$

$$\frac{d}{dx}(\tanh u)=\operatorname{sech}^2 u\frac{du}{dx}$$

INTEGRALES

$$\int adx=a\int dx=ax+c$$

$$\int af(x)dx=a\int f(x)dx$$

$$\int(u\pm v\pm w\pm\ldots)dx=\int udx\pm\int vdx\pm\int wdx\pm\ldots$$

$$\int u^n du=\frac{u^{n+1}}{n+1}+c,\ n\neq -1$$

$$\int\frac{du}{u}=\ln|u|+c$$

$$\int e^u du=e^u+c$$

$$\int a^u du=\frac{a^u}{\ln a}+c\quad\begin{cases}a>0\\a\neq 1\end{cases}$$

$$\int udv=uv-\int vdu$$ (integración por partes)

$$\int ua^u du=\frac{a^u}{\ln a}\left(u-\frac{1}{\ln a}\right)+c$$

$$\int ue^u du=e^u(u-1)+c$$

INTEGRALES CON LOGARITMOS

$$\int\ln u du=u\ln u-u+c=u(\ln u-1)+c$$

$$\int\log_a u du=\frac{1}{\ln a}(u\ln u-u)+c=\frac{u}{\ln a}(\ln u-1)+c$$

$$\int u\log_a u du=\frac{u^2}{4}(2\log_a u-1)+c$$

$$\int u\ln u du=\frac{u^2}{4}(2\ln u-1)+c$$

$$\int u^n\ln u du=\frac{u^{n+1}}{(n+1)^2}\left[(n+1)\ln u-1\right]+c$$

$$\int\frac{1}{u\ln u}du=\ln|\ln u|+c$$

INTEGRALES DE FUNCIONES TRIGONOMÉTRICAS

$$\int\operatorname{sen} u du=-\cos u+c$$

$$\int\cos u du=\operatorname{sen} u+c$$

$$\int\tan u du=-\ln\cos u+c=\ln\sec u+c$$

$$\int\cot u du=\ln\operatorname{sen} u+c$$

$$\int\sec u du=\ln(\sec u+\tan u)+c$$

$$\int\csc u du=\ln(\csc u-\cot u)+c$$

$$\int\operatorname{sen}^2 u du=\frac{u}{2}-\frac{1}{4}\operatorname{sen} 2u+c$$

$$\int\cos^2 u du=\frac{u}{2}+\frac{1}{4}\operatorname{sen} 2u+c$$

$$\int\tan^2 u du=\tan u-u+c$$

$$\int\cot^2 u du=-(\cot u+u)+c$$

$$\int\sec^2 u du=\tan u+c$$

$$\int\csc^2 u du=-\cot u+c$$

$$\int u\operatorname{sen} u du=\operatorname{sen} u-u\cos u+c$$

$$\int u\cos u du=\cos u+u\operatorname{sen} u+c$$

$$\int\sec u\tan u du=\sec u+c$$

$$\int\csc u\cot u du=-\csc u+c$$

$$\int e^{au}\operatorname{sen} bu du=\frac{e^{au}}{a^2+b^2}(a\operatorname{sen} bu-b\cos bu)+c$$

$$\int e^{au}\cos bu du=\frac{e^{au}}{a^2+b^2}(a\cos bu+b\operatorname{sen} bu)+c$$

INTEGRALES DE FUNCIONES TRIGONOMÉTRICAS INVERSAS

$$\int\operatorname{arcsen} u du=u\operatorname{arcsen} u+\sqrt{1-u^2}+c$$

$$\int\arccos u du=u\arccos u-\sqrt{1-u^2}+c$$

$$\int\arctan u du=u\arctan u+\ln\sqrt{1+u^2}+c$$

$$\int\operatorname{arccot} u du=u\operatorname{arccot} u-\ln\sqrt{1+u^2}+c$$

$$\int\operatorname{arcsec} u du=u\operatorname{arcsec} u-\ln\left(u+\sqrt{u^2-1}\right)+c$$

$$=u\operatorname{arcsec} u-\operatorname{arccosh} u+c$$

$$\int\operatorname{arccsc} u du=u\operatorname{arccsc} u+\ln\left(u+\sqrt{u^2-1}\right)+c$$

$$=u\operatorname{arccsc} u+\operatorname{arccosh} u+c$$

INTEGRALES DE FUNCIONES HIPERBÓLICAS

$$\int\operatorname{senh} u du=\cosh u+c$$

$$\int\cosh u du=\operatorname{senh} u+c$$

$$\int\tanh u du=\ln\cosh u+c$$

$$\int\operatorname{csch}^2 u du=-\coth u+c$$

$$\int\operatorname{sech} u\tanh u=-\operatorname{sech} u+c$$

$$\int\operatorname{cschu}\coth u=-\operatorname{csch} u+c$$

INTEGRALES DE FRACCIONES

$$\int\frac{du}{u^2+a^2}=\frac{1}{a}\arctan\frac{u}{a}+c$$

$$\int\frac{du}{u^2-a^2}=\frac{1}{2a}\ln\frac{u-a}{u+a}+c;\quad\left(u^2>a^2\right)$$

$$\int\frac{du}{a^2-u^2}=\frac{1}{2a}\ln\frac{u+a}{u-a}+c;\quad\left(u^2<a^2\right)$$

$$\int\frac{udu}{a+bu}=\frac{1}{b^2}\left(a+bu-a\ln|a+bu|\right)+c$$

$$\int\frac{du}{u(a+bu)}=\frac{1}{a}\ln\left|\frac{u}{a+bu}\right|+c$$

INTEGRALES CON $\sqrt{\ }$

$$\int\frac{du}{\sqrt{a^2-u^2}}=\operatorname{arcsen}\frac{u}{a}+c$$

$$\int\frac{du}{\sqrt{u^2\pm a^2}}=\ln\left(u+\sqrt{u^2\pm a^2}\right)+c$$

$$\int\frac{du}{u\sqrt{a^2+u^2}}=\frac{1}{a}\ln\left|\frac{u}{a+\sqrt{a^2\pm u^2}}\right|+c$$

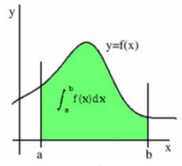
$$\int\frac{du}{u\sqrt{u^2-a^2}}=\frac{1}{a}\arccos\frac{a}{u}+c$$

$$\int\sqrt{a^2-u^2}du=\frac{u}{2}\sqrt{a^2-u^2}+\frac{a^2}{2}\operatorname{arcsen}\frac{u}{a}+c$$

$$\int\sqrt{u^2\pm a^2}du$$

$$=\frac{u}{2}\sqrt{u^2\pm a^2}\pm\frac{a^2}{2}\ln\left(u+\sqrt{u^2\pm a^2}\right)+c$$

PROPIEDADES DE LAS INTEGRALES DEFINIDAS



$$\int_a^b\{f(x)\pm g(x)\}dx=\int_a^b f(x)dx\pm\int_a^b g(x)dx$$

$$\int_a^b cf(x)dx=c\int_a^b f(x)dx\qquad c\in\mathbb{R}$$

$$\int_a^b f(x)dx=\int_a^c f(x)dx+\int_c^b f(x)dx$$

$$\int_a^b f(x)dx=-\int_b^a f(x)dx$$

$$\int_a^a f(x)dx=0$$

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